

## Erratum

Volume **85**, Number 2 (1982), in the article, “On a Problem of B. D. Sleeman,” by Paul Binding, pp. 291–307:

The arguments of Lemma 4.1 and Corollary 6.2 are incomplete, although they suffice if one makes the remarks after Lemma 4.2 and Corollary 6.2 into hypotheses. Specifically, we make the extra assumption:

(E)  $\Delta$  and  $\Gamma_n$  have unique selfadjoint extensions,  $1 \leq n \leq k$ .

Incidentally, Lemma 5.1 shows that (1.6) guarantees existence of the extensions, but makes no assertion about uniqueness. On the other hand, each of the conditions listed at the end of Section 5 suffices for (E). (For right definiteness, this is shown by Volkmer [4], while for bounded  $T_m$ , self-adjointness follows because the operators are everywhere defined. For left definiteness, we may assume without loss of generality

$$\alpha I \leq \Delta_{omn} \leq \beta I, \quad 1 \leq m, n \leq k \quad (1)$$

for some positive  $\alpha$  and  $\beta$  [1, Theorem 3.1]. Thus we can choose  $\alpha = (0, \dots, 0, 1)$ , whence  $\Delta = \Delta_k$  is essentially self-adjoint [1, remark after Theorem 3.1]. Moreover, if we write

$$E = \sum_{m=1}^k T_m^\dagger,$$

$$\alpha(x, Ex) \leq (x, \Delta_m x) \leq \beta(x, Ex), \quad 1 \leq m \leq k, \quad x \in \mathcal{D},$$

follows from (1). Therefore

$$[x, \Gamma_m x] / [x, x] = (x, \Delta_m x) / (x, \Delta_k x)$$

lies between  $\alpha/\beta$  and  $\beta/\alpha$ , so  $\Gamma_m$  is bounded on the dense subset  $\mathcal{D}$  of  $H_\Delta$ , and the extension is then by continuity.)

Thus while our results fall somewhat short of what Professor Sleeman intended, they do provide a unified approach to all the strongly definite problems in the literature. It is perhaps appropriate to point out some recent developments concerning our assumptions. It is shown in [2, Theorem 7.5] that  $\Delta \geq 0$  need be checked only on decomposable tensors, i.e., via a real-valued determinant depending directly on the original data. Moreover, in the situation of Corollary 8.3 (with  $T_m^{-1}$  compact), a complete orthonormal basis for  $K_\Delta^\perp$  has been constructed without assumption (E) [3, Theorem 5.3]. This adds interest to the question of when (or whether) (E) is actually necessary.

## REFERENCES

1. P. A. BINDING, Left definite multiparameter eigenvalue problems, *Trans. Amer. Math. Soc.* **272** (1982), 475–486.
2. P. A. BINDING, Multiparameter definiteness conditions II, *Proc. Roy. Soc. Edinburgh*, to appear.
3. P. A. BINDING, A. KÄLLSTRÖM, AND B. D. SLEEMAN, An abstract multiparameter spectral theory, *Proc. Roy. Soc. Edinburgh*, to appear.
4. H. VOLKMER, On multiparameter theory, *J. Math. Anal. Appl.* **86** (1982), 44–53.

PAUL BINDING